Quantum-information conservation. The problem about "hidden variables", or the "conservation of energy conservation" in quantum mechanics:
A historical lesson for future discoveries

Vasil Penchev, vasildinev@gmail.com
Bulgarian Academy of Sciences: Institute of Philosophy and Sociology:
Dept. of Logic and Philosophy of Science

Abstract. *The explicit history of the "hidden variables" problem is well-known and established. The main events of its chronology are traced. An implicit context of that history is suggested. It links the problem with the "conservation of energy conservation" in quantum mechanics. Bohr, Kramers, and Slaters (1924) admitted its violation being due to the "fourth Heisenberg uncertainty", that of energy in relation to time. Wolfgang Pauli rejected the conjecture and even forecast the existence of a new and unknown then elementary particle, neutrino, on the ground of energy conservation in quantum mechanics, afterwards confirmed experimentally. Bohr recognized his defeat and Pauli's truth: the paradigm of elementary particles (furthermore underlying the Standard model) dominates nowadays. However, the reason of energy conservation in quantum mechanics is quite different from that in classical mechanics (the Lie group of all translations in time). Even more, if the reason was the latter, Bohr, Cramers, and Slatters's argument would be valid. The link between the "conservation of energy conservation" and the problem of hidden variables is the following: the former is equivalent to their absence. The same can be verified historically by the unification of Heisenberg's matrix mechanics and Schrödinger's wave mechanics in the contemporary quantum mechanics by means of the separable complex Hilbert space. The Heisenberg version relies on the vector interpretation of Hilbert space, and the Schrödinger one, on the wave-function interpretation. However the both are equivalent to each other only under the additional condition that a certain well-ordering is equivalent to the corresponding ordinal number (as in Neumann's definition of "ordinal number"). The same condition interpreted in the proper terms of quantum mechanics means its "unitarity", therefore the "conservation of energy conservation". In other words, the "conservation of energy conservation" is postulated in the foundations of quantum mechanics by means of the concept of the separable complex Hilbert space, which furthermore is equivalent to postulating the absence of hidden variables in quantum mechanics (directly deducible from the properties of that Hilbert space). Further, the lesson of that unification (of Heisenberg's approach and Schrödinger's version) can be directly interpreted in terms of the unification of general relativity and quantum mechanics in the cherished "quantum gravity" as well as a "manual" of how one can do this considering them as isomorphic to each other in a new mathematical structure corresponding to quantum information. Even more, the condition of the unification is analogical to that in the historical precedent of the unifying mathematical structure (namely the separable complex Hilbert space of quantum mechanics) and consists in the class of equivalence of any smooth deformations of the pseudo-Riemannian space of general relativity: each element of that class is a wave function and vice versa as well. Thus, quantum mechanics can be considered as a "thermodynamic version" of general relativity, after which the universe is observed as if "outside" (similarly to a phenomenological thermodynamic system observable only "outside" as a whole). The statistical approach to that "phenomenological thermodynamics" of quantum mechanics implies Gibbs classes of equivalence of all states of the universe, furthermore representable in Boltzmann's manner implying general relativity properly … The meta-lesson is that the historical lesson can serve for future discoveries.*

Key words: *BKS theory. class of equivalence, energy conservation in quantum mechanics, fourth uncertainty relation, general relativity and quantum gravity, Gibbs and Boltzmann thermodynamics, Heisenberg's matrix mechanics, pseudo-Riemannian space, Schrödinger's wave mechanics, separable complex Hilbert space, unitarity*

A contextual definition of the "hidden variables" problem in its "early history":

"Early history" means the period from the beginning of the thirties to the beginning of the eighties of the twenty century. It comprises the following stages according to the most important and most cited publications:

- Its explicit formulation: Neumann (1932); Einstein, Podolsky, and Rosen (1935); Bohr (1935); Schrödinger (1935)
- Neumann's theorem (1932: 167-173) about the absence of hidden variables in quantum mechanics
- The articulation of the ways for experimental test: Bell (1964); Clauser, Horn, Shimony, Holt (1969)
- Kochen and Specker's theorem (1968) about the absence of hidden variables in quantum mechanics
- The first experiments convincingly confirming that absence: Clauser, Horn (1974); Aspect, Grangier, and Roger (1981 and 1982)

A short formulation of the problem can be the following:

The mathematical formalism of quantum mechanics based on the complex Hilbert space needs and requires only the half variables in comparison with the exhausted description of the same mechanical system in classical mechanics. What is the physical meaning of the other half of variables seeming "hidden" in quantum mechanics?

The concise description of the meaning and influence of the problem in history of philosophy of science can be the following. It concentrates in a single scientific problem a series of philosophical problems in quantum mechanics, physics, and science about:

- Determinism
- Holism
- Measurement
- The relation of model and reality
- Space-time and energy-matter universality
- The locality (or globality) and separability (or inseparability) of the apparatus and researched object
- Information as a philosophical concept as a physical ground of the universe
- Too many others

Even more, that single scientific problem could be experimentally resolved and thus this assists indirectly the elucidation of those problems in philosophy of science. The problem has been exceptionally widely discussed in philosophical literature.

The meaning of the problem in history of quantum mechanics can be described as follows. It generates a new and booming physical discipline: (theory of) quantum information studying the phenomena of entanglement and quantum correlations, the physical existence of which is implied by the absence of hidden variables in quantum mechanics (or as a special kind of nonlocal hidden variables in it) as well as an interdisciplinary area of the same name studying and developing the mathematical formalism underlying the former and its implementation in other domains. The theory of quantum information allows for a complete informational reinterpretation of quantum mechanics and by means of it, of other physical and chemical disciplines and its technical application by the theory of quantum computer.

Prehistory, background, and context

The quantities necessary to be described any mechanical system in the quantum case are the exact half to those in the classical case, and the missing "other half" can be considered as unknown, but existing "hidden variables" once added to the former half, they would describe the quantum-mechanical system "completely". This is the "hidden variables" conjecture with Einstein as its most famous convinced proponent.

An analogy to the relation of phenomenological to statistical thermodynamics is added often in the framework of the conjecture:

The latter demonstrates the former as classes of equivalence sharing the same probability distribution in certain mechanical quantities and resulting in the same set of phenomenological thermodynamic quantities. In other words, many microscopic mechanical descriptions share the same single macroscopic phenomenological thermodynamic description. Only the number of the mechanical particles in any certain state does matter rather than exactly which particles share it.

A generalized mathematical model suggests the class of equivalence of all well-orderings relevant to a single certain ordinal number. The well-ordering "theorem" as well as the axiom of choice are involved implicitly in it: thus, the quantity of information as the amount of elementary *choices* in the units of bits.

The "hidden variables" conjecture admits that an analogical relation between quantum mechanics and classical mechanics:

Quantum mechanics studies only the classes of the equivalence of all possible well-orderings (e.g. as Feynman's virtual pathways) to a certain quantum leap for the fundamental quantum indistinguishability of the elements of any quantum-mechanical system. Anyway, the other half of quantities (thus describing unambiguously any certain well-ordering in the framework of a certain ordinal number) could be added explicitly is what the conjecture for hidden variables states.

However, many real experiments starting from the ideas of Einstein, Podolsky, and Rosen's *Gedankenexperiment* (1935) and Bell's theorem (1964) reject the conjecture, and thus, they confirm the "completeness" of quantum mechanics (by Einstein, Podolsky, and Rosen's term).

Consequently, quantum mechanics means those classes of equivalences unlike classical mechanics studying both classes and elements of them. This can be visualized by the fundamental postulate formulated by Niels Bohr:

Quantum mechanics studies quantum entities only in the system of the investigated microscopic quantum entities and the measuring apparatus macroscopic, and thus, described by the smooth differential equations of classical mechanics, exceptionally by the readings of the apparatus.

Furthermore, all experimental science (i.e. including quantum mechanics) relies on the fundamental epistemological postulate for science to be objective, and particularly, the influence of the measuring apparatus in experimental science to be negligible and grantable as zero. The direct physical influence of the macroscopic apparatus to the also macroscopic entity investigated in classical physics is both relatively small, reducible under a maximally admissible limit, and controllable. Thus, it can satisfy practically the epistemological obligation to be considerable as zero. However, the same approach is fundamentally inapplicable in quantum mechanics for the quantities of the studied entities are practically "zero" to those of the apparatus itself.

An alternative approach utilized by quantum mechanics consists in the necessary epistemological "transparence" of the apparatus to be postulated (as in the separable complex Hilbert space of quantum mechanics) by means of the class of equivalence of all results to any possible apparatus, still more, being consistent or even identical to the fundamental discreteness of quantum mechanics, also forced by the Planck constant.

Particularly, the same implies that the apparatus in quantum mechanics is not "almost" transparent, but absolutely (and even, "identically") transparent for this is postulated as a necessary condition of its mathematical apparatus, and for this, deducible from a few properties of the separable complex Hilbert space as the well-known theorems of the absence of "hidden variables". Indeed, any hidden variable suggests in definition to be a complement of the real state to the observed state. If they coincide identically as the mathematical formalism of quantum mechanics requires necessarily, this implies no hidden variables, in fact.

This means a revolution in the domain of experimental epistemology, to which quantum mechanics belongs as an experimental science, comparable to Kant's "Copernican revolution" in ontology by transcendentalism as it postulates a special kind of ontology, "transcendental", which can be interpreted also as featured by the zero epistemological difference as a necessary condition for cognition.

On the contrary, all classical science accepted the same difference as nonzero, therefore justifying the cognition itself by gradually reducing and diminishing that difference in the process of cognition, particularly, by more and more precisions mathematical models. That "opportunistic" permanent progress, however, turns out to be redundant if the ideal of the zero "model – reality" difference is reached as both transcendentalism speculative and quantum mechanics experimental claim.

The same class of invariance to all possible apparatuses implies energy conservation by the reason fundamentally different and even seemingly inconsistent to that in classical mechanics. The global Lie group (smooth) of translations in time underlies energy conservation in the latter according to Emmy Noether's first theorem (1918). The same group cannot exist in the discrete quantum mechanics fundamentally.

As Wolfgang Pauli stated, time unlike all other physical quantities is "only a number" (i.e. not an operator in the separable complex Hilbert space as all the other physical quantities). This means that time as a physical quantity refers only to classical physics and thus, only to the apparatus in the framework of quantum mechanics.

The following consideration can visualize time as "only a number". Any fundamentally random choice after quantum measurement refers to only one single course of time unambiguously associable to the apparatus used, as "only a number". However, the quantum state by itself, being a wave function, is independent of which apparatus is utilized, or it is invariant to the set of all possible apparatuses measuring the same quantum state. That invariance implies that no time operator is possible other than trivial identity, thus meaningless, as well as energy conservation; the latter as follows:

The trivial time operator being the trivial identity implies energy conservation by a reason quite different than Emmy Noether's argument in her first theorem (1918), but equivalent to it in virtue of the equivalence of the well-ordering "theorem" and the axiom of choice. One can notice that the latter equivalence is able to equate a smooth Lie group to a discrete symmetry, therefore implying Noether's approach. In other words, that "choice-ordering" equivalence in turn can be generalized as a generalization of her theorems of conservation, furthermore, necessary for inferring energy conservation in quantum mechanics (being discrete).

Indeed, the axiom of choice states that any element in any set can be chosen. Thus, it implies a derivative set of sets, each of which consists of a single element (chosen) of the former set. Furthermore, the former set and the latter set share the same certain ordinal number. That ordinal number is the class of equivalence of all possible well-orderings as of the latter as of the former set. The bijection of a well-ordering into another defines a permutation therefore being discrete.

Emmy Noether's theorems mean Lie groups only in a certain single trajectory, but not any discrete transformation between different trajectories (properly suggesting a quantum leap). However, the generalization of the choice-ordering equivalence can be considered as a generalization implied by the theorems if they are applied to themselves. In other words, if the theorems are meant as defining implicitly, but unambiguously, Lie groups different from those Lie groups meant in their formulation, the theorems in relation to those Lie groups of a meta-level imply symmetries (eventually parametrized as in the second theorem).

In other words, one can applies the theorems to a trajectory consisting of trajectories as in the Feynman interpretation of wave function. That generalization also relevant to the choice-ordering equivalence implies energy conservation in quantum mechanics representing the class of equivalence of all well-orderings (to each of which a certain course of time in a possible apparatus refers unambiguously as Pauli's "only a number") as an ordinal number. That class of equivalence defines a symmetry referring to the Lie groups relevant to the corresponding wave function (being smooth) in virtue of Noether's theorems.

In fact, that energy conservation in quantum mechanics (for a reason different from that in classical mechanics) is postulated in the separable complex Hilbert space as unifying Heisenberg's matrix mechanics and Schrödinger's wave mechanics. Here is how:

Heisenberg's matrix interpretation means the vector interpretation of that Hilbert space unlike Schrödinger's wave mechanics meaning the function interpretation of the same. Thus, a necessary condition of the unification of Heisenberg's and Schrödinger' versions is the equivalence of both interpretations of Hilbert space (particularly, as to the separable complex Hilbert space of quantum mechanics).

In turn, a necessary condition of the above necessary condition is the class of equivalence of all unitary transformations of any vector for equating the well-ordered vectors to functions, which are not ordered because of the commutativity of the additive members of any function. The postulation of that class of equivalence, necessary to be equated both interpretations of Hilbert space, interpreted in terms of quantum mechanics, in turn, implies energy conservation in quantum mechanics.

In other words, if one uses the model of the separable complex Hilbert space (as the standard contemporary quantum mechanics does) energy conservation is involved implicitly. However, it is not a corollary from smooth time translations as in classical mechanics since those translations are irrelevant to quantum mechanics.

The physical and philosophical interpretation of the above mathematical condition is the following:

The energy of any quantum entities does not depend on which apparatus has measured it. It is a corollary from the invariance to all possible apparatuses, by which the energy of quantum system can be measured. An own time can be associated with any possible apparatus, and then a relevant "meta-time" (smooth, but reversible as in both special and general relativity) can be assigned to the set of all those own times of the apparatuses or in each of all Feynman pathways. Thus, it would imply the conservation of wave function: that symmetry being correlative to the meta-time in virtue of the first Noether theorem.

That conservation of wave function rearticulated in terms of quantum information can be called "quantum information conservation". In turn, energy conservation in quantum mechanics is a corollary from quantum-information conservation rather than from the principle of least action after that principle is not valid literally in quantum mechanics.

Furthermore, quantum-information conservation is a much more general law rather than energy conservation in quantum mechanics. For example, the transition between any microscopic quantum entity and the macroscopic apparatus does not conserve energy (even the exponents of their energies are quite different), but conserves wave function, i.e. quantum information. If the corresponding quantity of quantum information was not conserved, even the quantum cognition itself would be fundamentally impossible.

Furthermore, quantum-information conservation implies a very interesting generalization of conservation in the transition between apparatuses (surprisingly relevant to the generalization of energy conservation in general relativity). It will be discussed in detail further, but it is linked to Bohr, Kramers, and Slater's conjecture (1924) for violation of energy conservation in quantum mechanics because of the "fourth Heisenberg uncertainty":

The energy-time uncertainty implies that if energy is conserved and thus, constant, time is absolutely uncertain; and vice versa as well. The only question is how the quantities of energy and time to be interpreted, more precisely, the time or energy of what is meant:

1. Energy conservation and time uncertainty. The consideration above elucidates that a specific time is assignable to each possible apparatus able to measure out that energy constant for conservation. Then, the absolute uncertainty of time means an absolute uncertainty of which apparatus has measured; or in other words, the invariance to the set of all possible apparatuses meant by energy conservation in quantum mechanics.

2. Energy uncertainty and time "conservation". This mean that a single certain apparatus has been chosen, but quantum complementarity implies that what (which quantum entity) has been measured is absolutely unknown in principle. Thus, the energy of what (being whatever) has been absolutely uncertain. The BKS theory means that latter case. In fact, this is out of the scope of quantum mechanics if Bohr's postulate what quantum mechanics studies is valid: indeed, it studies quantum entities (which is relevant to the former case) rather than apparatuses (which is relevant to the latter case).[1]

---

[1] However the latter case meant by the BKS theory implicitly is absolutely relevant to the conjecture that general relativity can be considered as the ultimate theory of … quantum gravity under a certain condition. That consideration will be discussed in detail further in the paper.

This is the essence of Pauli's contra-argument accepted absolutely by Bohr and resulted in the conjecture of then hypothetical "neutrino" as well as in the dominating paradigm of "elementary particles", in the framework of which the Standard model is situated.

Then, if quantum gravity needs the latter case (as this will be demonstrated bellow), the Standard model (as needing the former case) would be inconsistent to quantum gravity fundamentally. Even more, the kind of that inconsistency would be in the sense of quantum complementarity.

Both matrix and wave mechanics can be reinterpreted reversely on the base of their unification as two complimentary interpretations each of which is referable either to the investigated microscopic quantum system or to the apparatus, but not to both simultaneously. Quantum mechanics by the separable complex Hilbert space unifying Heisenberg's matrices and Schrödinger's wave functions is only what is able to mean both simultaneously. Here are those reverse reinterpretations in detail:

1. The standard viewpoint (i.e. the conservation of energy conservation unlike the BKS theory) suggests the discrete matrix mechanics to be assigned to the investigated quantum system "by itself", and the smooth wave mechanics, to its image by the readingc of the apparatus. The fundamental epistemological postulate for cognition to be "transparent" equates them. However, their equating is postulated, furthermore, in the mathematical formalism of quantum mechanics, and thus, implies the theorems of the absence of hidden variables in it: in other words, the epistemological postulate (externally added in classical mechanics, physics, and experimental science) is written and embodied in the mathematical model itself as its necessary condition. Thus, that kind of mathematical model is able to supply an internal proof of consistent completeness (what the theorem of the absence of hidden variables are). What follows?

Heisenberg's mechanics describes the quantum system at issue by itself. Schrödinger's mechanics does the same in terms of the apparatus being empirically accessible for humans. The epistemological postulate embodied in the separable complex Hilbert space equates them therefore implying as a necessary condition the conservation of energy conservation.

This is the standard viewpoint in quantum mechanics, generalized as the "paradigm of elementary particles", and embodied in the Standard model. However, it is non-standard (or more precisely, "anti-standard") from the rigorous mathematical meaning of the Lövenheim – Skolem theorem (more precisely, Skolem's argument (1922) more known as Skolem's "paradox" though not being paradoxical in fact). It is "anti-standard" in the following meaning:

A bijection is able to build a smooth (and thus, continuous) set (meant in wave mechanics) as a mapping of a discrete set (meant in matrix mechanics), that is, reversely, to the sense of "nonstandard interpretation" which demonstrates an unambiguous discrete set as a mapping of a continuous set. As far as the mapping is a bijection, each of them implies the other one, and the difference between them is only as in Frege's "Sinn" rather than as in his "Bedeuting".

2. The BKS theory allows for a nonstandard interpretation of quantum mechanics, therefore being the standard counterpart in the sense of set theory. Instead of a certain quantum system investigated by all possible apparatus as in "1." above, one considers a certain apparatus which registers all possible quantum systems, thus with any energy (the case of not conserving energy conservation). The discrete matrix mechanics is assigned again to the quantum system(s), and the smooth wave mechanics, to the apparatus meaning the epistemological situation in "1.".

However Bohr's postulate about what quantum mechanics studies is modified. That nonstandard quantum mechanics studies the apparatus by means of its readings corresponding to each possible quantum system registerable by it. As far as the separable complex Hilbert space is granted as a relevant mathematical formalism in the case as well, the epistemological postulate is valid still. Then, the apparatus would be described by the class of readings to all possible quantum systems separated in disjunctive subclasses, each of which correspond to a certain quantum system.

One may notice that formal and mathematical structure of the "investigated" single apparatus in "2." is analogical (and possible, even the same as a conjecture) to that in the Standard model: a global space of the apparatus itself, each point of which is a local space assignable to each certain quantum system among all possible.

Then, each local space is described in terms of matrix mechanics (meaning "2.") or in terms of quantum mechanics, but they are equivalent once the latter has been involved. Thus, one can speak of a single local space exemplified identically in each point of the global space.

One can consider the apparatus from two different viewpoints possibly equivalent to each other: that of the smooth classical mechanics or that of the also smooth wave mechanics:

A. How the apparatus can be described by classical mechanics.

B. How the apparatus can be described by wave mechanics.

A. The set of all quantum systems (with different energies and momenta) measurable by the apparatus investigated should be situated in a smooth way (as the smooth classical mechanics needs) in a mathematical space.

Then, an additional postulate can be added: that mathematical space is the physical space-time according to general relativity, and thus, pseudo-Riemannian space or a relevant compact subspace of it, e.g. that corresponding to the space-time area occupied by the apparatus. All quantum systems measurable by the apparatus turn out to be ordered well in four independent variables corresponding to space-time. The condition of smoothness implies that any of those four independent variables can be used equivalently for any small enough, i.e. infinitesimal area. Any of them is necessary to be added as an independent additional dimension of the well-ordering only after a finite distance in it. Meaning this, one can order well the points of all the space-time, dimension by dimension. The three-dimensional smooth areas are ordered as points in the fourth dimension, the two-dimensional smooth areas are ordered well in the third dimension in each three-dimensional smooth area, the one-dimensional areas are ordered well in the second dimension in each two-dimensional area, the (zero-dimensional) points are ordered in the only (first) dimension in each one-dimensional area. The procedure of the well-ordering of space-time is finished. All quantum systems measurable by the apparatus are ordered well also, in virtue of the postulate bijection of space-time into them.

One can consider the class of equivalence of all those possible well-orderings, respectively the corresponding ordinal number associable to the apparatus investigated.

An equivalent gravitational field corresponding to the apparatus would be constructed so. Each certain well-ordering of the space-time of the apparatus would be a possible state of it. Being a gravitational field, it could described exhaustively by the Einstein field equation of general relativity.

And vice versa, meaning a certain gravitational field described by the Einstein field equation, it can be considered equivalently as describing a hypothetical apparatus under the condition of the nonstandard interpretation of quantum mechanics.

Energy conservation would not be valid in its framework (unlike in that of the standard interpretation of quantum mechanics). As in general relativity, a generalized conservation law, namely energy-momenta conservation would be valid in the transition from a quantum system to another, being equivalent to the transition from a point to another in the corresponding gravitational field.

One can discuss the postulate allowing for that result to be inferred: the mathematical space, in which all quantum systems measurable by the apparatus at issue are to be situated unambiguously, is space-time or a certain subspace of it. Any apparatus determines a certain space-time area, in which it is situated. One does not mean the real gravitational field corresponding to that space-time area.

The procedure described above assigns a new, absolutely different gravitational field of the apparatus is granted as the transcendental totality (in philosophical terms) or as a holistic whole (in terms of the philosophy of quantum mechanics. If one identifies the former (as the universe) and the latter, our single universe can be considered as an apparatus therefore involving a certain quantum-gravitational filed as the own gravitational field of the universe described by the Einstein field equation. This means that quantum gravity is usual gravity under the condition for the apparatus to be granted as a universe. The justification for the condition to be involved is the identification by the totality (the totality being just total can be accepted as a single one therefore generating many hypostases within itself by itself).

If one means the absolute totality of the universe, the externality of which should be within it in definition, the equivalent mirror representation of the same is the absolute totality of the quantum, the internality of which should be out of it in definition. Speaking figuratively, what the universe is, is the internality of a quantum (whether a certain quantum or any quantum meaning their indistinguishability). Analogically, the transition between quantum gravity (as a nonstandard interpretation of quantum mechanics) and gravity (as in general relativity) is the transition between two equivalent descriptions, the former quantum, external, “outside”, and the latter classical, internal, ‘inside”.

The transition from special to general relativity as a generalization of invariance to reference frames: from all inertial to all arbitrarily accelerated. Analogically, a transition to a new “quantum relativity” can be involved simultaneously by two new kinds of reference frames: discrete (quantum) reference frames and external reference frames, to which all reference frames of general relativity are internal. The former means a quantum leap between any two of those. The invariance to the latter implies the equivalence of quantum gravity and the gravity of general relativity

What the invariance to the former could mean will be discussed after the description of the nonstandard quantum mechanics of the apparatus in terms of Schrödinger’s wave mechanics as in “B.”.

One can notice that the class of equivalence of all well-orderings (mathematically meant as an ordinal number) is utilized in two independent fundamental conceptions of the contemporary physics: statistical thermodynamics and conservation-symmetry laws after Noether’s theorem (1918); therefore suggesting some hidden or unknown link between them.

B. How the apparatus can be described by wave mechanics.

The apparatus is equivalent to a smooth quantum field, i.e. to a mapping of the Newtonian space and time of classical physics into the separable complex Hilbert space of quantum mechanics. All possible quantum systems measurable by the apparatus are represented as different points in Euclidean space by a consideration analogical to that in A. This is not any pseudo-Riemannian space because one means a certain wave function in each point of it rather than only the value of energy-momenta as in A, where space-time turns out to be curved to pseudo-Riemannian for to be both well-ordered and smooth[2].

In other words, the separable complex Hilbert space and pseudo-Riemannian represent the same in different ways. The same idea will be deepen further to their isomorphism under the double condition of (1) the class of equivalence of all well-orderings of the same set as its ordinal number and (2) the choice-ordering equivalence.

The quantum field corresponding to the apparatus in the nonstandard quantum mechanics describing it (rather than any quantum system) as any quantum field implies the idea of "gauge invariance" (shared by the Standard model as well). Then, the intended isomorphism of the separable complex Hilbert space and pseudo-Riemannian space allows for gauge invariance to be developed further by the identification of the global and local space as the same.

The idea of gauge invariance ascribes unambiguously a local space to (as if "within") any point of the global space, which can be considered as a "gauge field" able to identify all local spaces as the same in the condition of a gauge symmetry corresponding to that gauge filed. According to gauge invariance in the framework of the Standard model, the relevant gauge symmetry is $\{[U(0)]\otimes[SU(1)]\otimes[SU(2)]\}$ in the separable complex Hilbert space. Thus, the global and local space are identified as the same. The isomorphism of them is trivial and does not depend on any condition because is a tautology definitively.

One can generalize the same idea to "transcendental field" to be "gauging":

The "transcendental field" is a field, in which the local and global space are isomorphic under a certain condition called transcendental. The gauge field of the Standard model is a trivial transcendental field, for example.

A transcendental field of quantum gravity is to be involved as follows:

The separable complex Hilbert space of quantum mechanics and the gravitational pseudo-Riemannian space of general relativity are isomorphic under the transcendental condition of (1) the class of equivalence of all well-orderings of the same set as its ordinal number and (2) the choice-ordering equivalence.

The text above contains a demonstration of the unification of Heisenberg's matrix mechanics and Schrödinger's wave mechanics under that transcendental condition as necessary (where it implies both "conservation of energy conservation" and "no hidden variables" as to the case of the standard interpretation of quantum mechanics). Thus, it is necessarily available in the case of the transcendental field of quantum gravity. In other words, it implies an analogical unification of quantum mechanics and general relativity (as to the case of the "nonstandard interpretation" of quantum mechanics) as

[2] If it was Euclidean space always, this would imply for neighboring well-ordered points in it (i.e. sharing the same energy-momenta, with infinitesimally small difference eventually) to be in a finite distance from each other implying discrete topology. The same can be reflected in the case of quantum field (as in B.) by means of the fact any wave function to be complex (e.g. particularly, a complex constant): the curved space, in which the covariant and contravariant counterparts do not coincide in general, can be equivalent by a single complex vector, the imaginary part of which correspond to the one of them, and its real part, to the other counterpart. That representation is homeomorphism furthermore. Thus, quantum field involving as values the complex wave function is able to conserve the argument space not to be curved, just as Euclidean space is not curved.

well as a generalization of energy conservation, the identification of entanglement and gravity under the transcendental condition of quantum gravity, and a certain conjecture about the origin of dark matter (from the equivalent space-time curvature) and dark energy (from the equivalent energy-momenta curvature) from entanglement.

One need demonstrate the isomorphism[3] of the separable complex Hilbert space and pseudo-Riemannian space under the transcendental condition above:

The isomorphism can be divided into the composition of two independent sub-isomorphisms:

1. The isomorphism of the smooth structure of pseudo-Riemannian and the discrete structure of complex separable Hilbert space. The above transcendental condition is sufficient to supply that sub-isomorphism.

2. The isomorphism of an element of that smooth structure and an element of that discrete structure. That isomorphism is to be demonstrated, and it implies the units of quantum information, qubits. More generalized, the isomorphism of any smooth structure and any discrete structure implies the units of quantum information as the unification and identification of their elements. However, the latter will remain only a conjecture as to the framework of the present paper since that goes out of it.

That isomorphism can be demonstrated and visualized as follows:

A qubit is defined as the normed (as a unit) superposition of two orthogonal subspaces of the separable complex Hilbert space. As far as any two successive "axes" of it are those orthogonal subspaces particularly, it admits an equivalent representation by a series of qubits or by a set of the same qubits, each of which correspond to a pair of two successive axes.

Also, any qubit can be considered as an element as in the second sub-isomorphism above in virtue of the following:

Any qubit defined so is isomorphic to a unit sphere (or ball), and two points of it, which are chosen on two orthogonal great circles (orthodromes). The dual counterpart (being anti-isometric) of any qubit can be defined as an (anti-isometric) dual qubit. Then, there exists a nonstandard homeomorphism being due to the above transcendental condition of the separable complex Hilbert space and Minlowski space after which (e.g.) the two dual Hilbert spaces are mapped into the real and imaginary domain of Minkowski space correspondingly.

Next, if one interprets those two domains correspondingly as the covariant and contravariant projections of a pseudo-Riemannian space, the particular case of a "flat" pseudo-Riemannian space (corresponding to Euclidean space) would be obtained if both projections are identical.

If the pseudo-Riemannian space is not flat, the projections will not be equal (identical): let that be the case. Then, one obtains certain nonzero entanglement between certain qubits and their "dual" counterparts as far as they correspond to a pair of non-identical projections (contravariant and covariant) of pseudo-Riemannian space.

---

[3] One could notice that the same isomorphism would imply a certain link between the gauge symmetry $\{[U(0)]\otimes[SU(1)]\otimes[SU(2)]\}$ of the Standard model and the invariance to arbitrarily accelerated reference frames in general relativity as a specification of the dependence of invariance, symmetry, and conservation in physics as in Noether's theorems (1918). The gauge symmetry itself can be interpreted also abstractly and mathematically in the framework of a fundamental informational structure consisting of two qubits considered jointly as a single bit, thus implying a triple of qubits: $[U(0)]$ to each single qubit; $[SU(1)]$ to any pair of them; and $[SU(2)]$ to all the three; $[U(0)]\otimes[SU(1)]$ as well as $[U(0)]\otimes[SU(2)]$ imply the Higgs mechanism, particularly.

However, the same certain state of entanglement allows for another, not “curved” representation, which is due to the infinite dimensionality of Hilbert space: all entangled qubits can be represented equivalently as a pair of non-entangled qubits.

In other words, the “transcendental space” (i.e. the identical local and global space) corresponding to the transcendental field of the Standard model, namely separable complex Hilbert space, is equivalently represented (by the isomorphism sketched above) into the space-time of general relativity, namely pseudo-Riemannian space, only under the implicit, self-evident condition the former to be relevant to quantum mechanics.

The same result can be reached also if one identifies the description of the apparatus by A. (in the framework of classical mechanics) to that by B. (in the framework of wave mechanics), both in the framework of the nonstandard of quantum mechanics. The former involves pseudo-Riemannian space to describe the apparatus, the later implies the separable complex Hilbert space as both local and global space as in the Standard model.

The identification of local and global space as transcendental space allows for a maximal generalization, only in terms of set theory by the following structure referring to a certain set. It introduces a well-ordering of all well-orderings of the set. Any well-ordering can be considered simultaneously as the corresponding ordinal number (as in Neumann’s definition of “ordinal number”) of the class of all well-orderings (as in Cantor – Russell’s definition of ordinal number).

That duality of well-ordering and ordinal number represents the set-theoretical ground of the “conservation of energy conservation” and the “absence of hidden variables” in quantum mechanics, and implies a generalization of energy conservation (from which the conservation of energy conservation is implied in quantum mechanics rather than from the principle of least action being invalid in it), namely quantum information conservation discussed a little above.

The ground of quantum information conservation in set theory is the following: the quantity of information in the set of all possible choices of a single event from a given set and that in any well-ordering of the given set is the same (that equality is especially heuristic in the case of an infinite given set, i.e. where the meant information is properly quantum).

In order words, quantum information conservation is invariant (or otherwise speaking, it remains the same) between a group and a corresponding symmetry as in Noether’s theorems, therefore involving a maximally generalized symmetry (at least as to the contemporary cognition or even at all, being transcendental in a physical and mathematical sense). The corresponding group is that of the quantity of physical action as this will be explained in detail soon.

Indeed, any bit of information means as the elementary choice among two equally probable alternatives (thus unordered) as any ordering of them (thus ordered). Then, the set-theoretical sense of (quantum) information conservation would mean an invariance to ordering (as a living room whether in any way or unordered at all), and the logical one, to any change of Frege’s “Sinn” and/ or “Bedeutung”.

Meaning the idea of information conservation, that set-theoretical transcendental structure can be described as follows. Any set implies it as its well-ordering, ordinal (“globally”) consisting of all well-orderings, ordinal (“locally”). Any local well-ordering can be named unambiguously by that well-ordering which is “immovable” under the transformation between “globally” and “locally”.

“Globally” and “locally” can be identified (and the identification to be called “transcendentally”) if and only if information conservation is valid.

This can suggest the transcendental meaning of information conservation: it is valid if and only if physical and mathematical transcendentality is involved.

The identification of the space-time of general relativity, i.e. pseudo-Riemannian space, and the transcendental space of the Standard model, i.e. the separable complex Hilbert space (under the transcendental condition) can suggest an idea about a possible origin of both dark matter and dark energy after the identification of gravity and entanglement in a way, which follows:

A preliminary notice is necessary. The ground of general relativity is as the equality of “gravitational mass” and “inertial mass” as the equal action of the gravitational field and the field caused by any other force different from gravitation. Let one admit that the inertial mass refers to certain elementary particles thus obeying the Standard model and influenced by a certain quantum field, accordingly. Both Standard model and quantum field (whatever) do not include gravitation as it is well-known. However, they would involve equal gravitational masses and filed(s) if both equalities above hold in quantum mechanics just as they hold in classical mechanics and general relativity.

If one admit that they hold universally and everywhere, this can be justified very easily by means of the concept of transcendental field and the identification of quantum mechanics and general relativity and their corresponding spaces. Both gravitational field and mass are due the smooth interpretation of the underlying physical and mathematical structure of quantum information just as the three other fundamental interactions within the framework of the Standard model and the corresponding inertial mass by the Higgs mechanism are the discrete quantum interpretation of the same structure.

The underlying property of the level of set theory involves the relation of the finiteness (e.g. by any natural number in Peano arithmetic) and actual infinity (e.g. by any infinite set in set theory). That relation is able to “flatten” any “curved space” being able to be extended and “stretched” unlimitedly, i.e. until the “curved space” turns out to be “flat”.

The same relation interpreted reversely allows for any arbitrarily curving starting from the “flat” actual infinity or the “flat” Standard model to be “curved” by “contracting” or “folding” from infinity (i.e. any actually infinite set) to any finiteness (i.e. to any natural number). One can admit infinite subsets (respectively subspaces), which share any nonempty even infinite intersection therefore effectively “curving”, being furthermore immediately visible after the nonstandard interpretation (e.g. of quantum mechanics) from the discrete and quantum into the smooth.

Particularly in terms of the separable complex Hilbert space of quantum mechanics, those infinite subspaces sharing a certain nonempty intersection would define equivalently “entanglement’, on the one hand, and their smooth nonstandard interpretation would result into a certain non-flat subspace of pseudo-Riemannian space implying a gravitational field being interpreted in terms of general relativity, on the other hand. Thus, the nonstandard interpretation of the discrete as the smooth or vice versa implies the *equivalence of entanglement and gravitation* as well as for general relativity to be interpreted as a “nonstandard” theory of quantum gravity. Then, its standard counterpart would be a relevant theory of entanglement (thus remaining a quantum theory sharing the concept of the separable complex Hilbert space).

That quantum theory of entanglement could elucidate the origin of both dark matter and dark energy as the two correlates in the Einstein field equation (space-time tensor and energy-momenta tensor, accordingly) in the gravitational field equivalent to entanglement as follows:

One considers any certain point of space-time, to which can be assigned mass and energy due to "non-dark" sources. Thus, space-time tensor and energy-momenta tensor in the point at issue are determined absolutely, and only one local space is chosen unambiguously by the "immovable point" of the global space in turn equivalent to the space-time point in question. That local space being a separable complex Hilbert space is "flat" and thus named all the class of "curved" counterparts, i.e. of all possible pairs, triples, tetrads, … (and so on) of entangled separable complex Hilbert spaces, each of which corresponds to an immovable point of the transcendental space. Tracing back to space-time, all class of those entangled local spaces will result into much more energy and mass (as far as both energy and gravity can be only positive) at the point at issue in comparison with that due to the non-dark sources and described by the Standard model.

Thus, entanglement being equivalent to gravity even in virtue of the equality of gravitational and inertial mass or that of gravitational field and any other field generating any acceleration to a unit mass at a point can be proved as a source of dark both mass and energy,

Even more, one can suggest verifiably that entanglement is the only source however after the following precision. Other sources of as dark matter as dark energy are possible in the framework of the Standard model (e.g. quarks by the mechanism of confinement) and thus out of the scope of general relativity or gravity, directly and explicitly. Nonetheless, all those non-dark sources can be equivalently represented by entanglement and then, by gravity, and thus, by general relativity, indirectly and implicitly.

In support of the said, one can add that all experimental facts in favor of dark mass-energy are due to astronomic observations and gravity, and thus in the framework of general relativity. There exists no one single corroborating or contradicting fact obtained from any other viewpoint or framework.

Thus, the concepts of dark matter and dark energy are underlain by "Mach's principle" as it was involved by Einstein (1918) in the paper introducing the "cosmological constant": only mass and energy are sources of gravitational field. In fact, those "dark" entities can be interpreted absolutely relevantly as the violation of Mach's principle, and thus, as the existence of some other source of gravitation.

The consideration above points out and named that third source of gravitational field, entanglement. In other words, as far as quantum information is the substance correlative to entanglement, one can generalized that there exists a third fundamental physical element besides mass and energy, namely quantum information or only information (at all):

As the famous equation of Einstein for the proportional equation of mass and energy is a corollary from the Lorentz invariance of special relativity, the crucial complement of (quantum) information is due to an analogical invariance of general relativity (equating it to quantum mechanics, particularly, as a nonstandard theory of quantum gravity) by the invariance of the transcendental field (introduced above), and thus, designable as transcendental invariance.

Transcendentalism understood physically and mathematically implies quantum information directly. Consequently, the transcendental field can be called quantum-information field also, in terms of physical substances such as mass and energy.

To a possible generalization of energy conservation in quantum mechanics: quantum information conservation

The text above demonstrated already that energy conservation in quantum mechanics can be inferred form a more general principle called "quantum information conservation" rather than from the principle of least action as in classical mechanics since the latter principle is not valid in quantum mechanics.

Now, a few other arguments, considerations and corollaries, as physical as mathematical, and as philosophical, from that generalized conservation will be discussed therefore outlining its wider context.

Energy conservation implies the slogan "The size does matter crucially!" For example, what has small energy can be equated physically to what has huge energy in no way. The small is small fundamentally and absolutely just as the big is big in the same way.

So, quantum mechanics studying microscopic entities is a theory quite different from general relativity referring to megascopic or at least to macroscopic entities. Not less wrongly, the scientific common sense suggests that the discreteness of quantum mechanics refers only to the microscopic elementary particles as far as the Planck constant is incredibly small. Thus the really granular quantum structure of the macroscopic (still more, of megascopic) physical objects is so exceptionally fine that it could not observed by the measuring devices. Their smooth mathematical models in classical physics, "ergo", are approximately true just as any mathematical model is approximate always and fundamentally. The discrete is discrete, the continuous is continuous just as "the small is small, and the big is big" is what the scientific common sense states implicitly, but unconditionally.

Well, this is no more than a prejudice is what quantum information conservation implies in an obvious challenge. For example, that prejudice consists in the 96% "darkness" of that kind of energy and matter in the contemporary physics. The origin of those is that prejudice unfortunately shared by all "normal" (in the sense of Thomas Kuhn), i.e. the official science. To overcome the "darkness", one need proclaim "revolutionarily" (again in his sense) that the big and small are relative just as the quantum and smooth or just as Einstein heralded that space and time are relative about a century ago.

The same viewpoint implies the following worldview. Speaking rather figuratively, the universe is situated within a single quant, in which it, itself is represent as all quanta available in the universe. That physical worldview seems to be too paradoxical and challenging common sense.

In fact, the universe is the greatest in physics just as a quantum is the tiniest. If one justifies the relativity of the small and big, of the microscopic and megascopic world(s), particularly, the same one can identify a quantum and the universe therefore involving a cyclic structure utilized a long time ago by Nicolaus of Cusa in ontology and theology. However, the implicit ontology of the Standard model rediscovered it absolutely independently by identifying the local and global space as the same separable complex Hilbert space. The concept of "transcendental field" and therefore "transcendental space" utilized here generalizes the latter complementing it by physical and mathematical transcendentalism implying pseudo-Riemannian space as the space-time of general relativity as the transcendental counterpart of the separable complex Hilbert space of quantum mechanics

In terms of general relativity being smooth the same involves the seemingly contradicting concept of "external reference frame" or (possibly equivalently) that of "discrete (or quantum) reference frames”. The invariance of the newly introduced reference frames to the standard ones (following the mental and intellectual tradition of general relativity as a heuristic principle) is able to conserve neither energy nor mass nor space-time, i.e. the “size” (speaking both literally and metaphorically), but only the quantity of quantum information, i.e. a wave function as a class of equivalence of certain space-time trajectories (called “world lines” in both theories of relativity).

The same invariance is available also in quantum mechanics as the “conservation of energy conservation”. The experimentally observed elementary particles cannot share energy with the apparatus being “incomparable in size”. So they share quantum information in both forms: of the particles themselves and by the readings of the apparatus. To be traced that invariance to the apparatus in detail, the nonstandard interpretation of quantum mechanics studying alternatively the apparatus rather than any elementary particles has been involved above, and therefore, an isomorphic mathematical structure describing the apparatus and satisfying the Einstein field equation for general relativity. Thus the invariance can be demonstrated as the same (whether identical or isomorphic) to the above description in terms of generalized reference frames.

The same kind of identification of a class equivalence by its element (for example taking place in Neumann’s definition of “ordinal number”) consistent to quantum information conservation and certain ways for it to be inferred is furthermore well-known a long time ago in physics, but in quite different branch of it, statistic thermodynamic, more precisely, in the relation of Boltzmann’s and Gibbs’s version. Thus, quantum information conservation relating general relativity and quantum mechanics to each other can be interpreted thermodynamically[4].

Quantum information conservation can be deduced properly mathematically from the equivalence of the separable complex Hilbert space and pseudo-Riemannian space. That mathematical meaning can be interpreted, speaking more loosely or philosophically, as the double or joint invariance of the straight and discrete, on the one hand, and the curved and continuous (smooth), on the other hand. It implies a symmetry on the former and latter therefore generalizing the Lie groups unifying them with infinite (and even finite, in the Dedekind, set-theoretical meaning) discrete groups. The conservation of quantum information as an only mathematical symmetry correlates just with that kind of generalized groups.

By means of philosophical reflection, the same symmetry can by interpreted as “transcendental”, being linked to the special definitive property of the totality to be all, or at least to that of the whole to be a new emergent entity unreducible thoroughly or unambiguously to its parts.

The whole (unlike the totality) as well as the relation to its parts is studied by all thermodynamic disciplines in physics featuring both phenomenological approach to the whole by itself and statistic approach to the whole as a function (fundamentally, not any bijection) of its parts. Thus, the ontological concept of the totality turns out to be partly investigated already, formally, physically, and mathematically, in the thermodynamic doctrine of a whole.

Quantum mechanics refers rather to the totality directly than to any whole. However, it possesses a few properties similar to a thermodynamic discipline, which is due to the historical facts and course,

[4] This implies furthermore and particularly, the interpretation of general relativity as a kind of thermodynamic or informationally-entropic theory, what Erik Verlinde’s theory of gravity is. The equivalence of the former and latter theory of gravitation under a certain and investigated condition is forthcoming to be deduced in a future work.

after which the totality is researched mathematically and physically, though partly, only thermodynamically, Indeed, the totality can be thought as the absolute and single totality including all, to which all the real physical thermodynamic wholes are particular, or the real correlates of the ideal essence or "idea" of the totality.

Once quantum information has been involved therefore the kind of unification shared in the class of quantum gravity, the same or analogical properties whether thermodynamic or informational-entropic, can be transferred and deuced in general relativity. Thus, it can be understood thermodynamically, informationally and entropically, and related to Erik Verlinde's theory of gravity.

Quantum information can be interpreted as the continuous link of mathematics and physics (corresponding invariantly to the standard discrete leap between those two scientific realms), a "smooth bridge" between them. It may be thought, more loosely and philosophically, as the quantity of how *many* mathematical to how *much* physical (by the mediation of geometry as a science both mathematical and physical), or how many mathematical structures per a unit of any physical quantity (or alternatively, per a unit of a generalized physical quantity such as "action"):

Thus and particularly, the fundamental Planck constant can be interpreted as how much physical action corresponds to a single mathematical unit as the most elementary mathematical structure, or in terms of the mediating concept of information: what is the physical (and thus material) action of an abstract and mathematical (and thus spiritual) bit of information. The transcendental boundary of the totality is what forces all ultimate unifications: both physical and mathematical, both spiritual and material; and quite particularly, but practically being exceptionally important, quantum mechanics and general relativity.

Consequently, the conservation of quantum information thought rather philosophically, is what allows for the smooth transition between the physical and mathematical, between what is material and what is spiritual, between gravity and all the rest, three interactions of the Standard model. It is able to transfer between the small and big, between the tiniest physically, a quantum, and the greatest, the universe, in a way for the "size" not to matter absolutely, thoroughly, and at all.

Quantum information as a physical quantity identical to its mathematical correlate has a Noether correlate of action itself. The conservation of quantum information in that meaning implies the physical group of action corresponding to that generalization and unification of Lie and discrete groups meant above as the only and properly mathematical meaning of quantum information conservation.

That physical group of action can be illustrated by its relation to the principle of least action and energy conservation in classical physics. In it and loosely speaking, it can be represented by the axiom that all "seconds" (i.e. any interval of time in the past, present, and future) are equal to each other and ordered in the only possible way. Furthermore, it is a smooth physical quantity, but unlike all other physical quantities, the "time arrow" makes sense.

Then, the physical group of action messes (thus making the "size" and energy meaningless being impossible to be defined unambiguously in general) all those postulated features of time in classical physics. For example, the "seconds" can be not equal (as in general relativity) or even not collinear being "curved" just as the rest three space dimensions can. They also can be unordered in any way (as in quantum mechanics), and time is neither smooth nor even a physical quantity at all (as in any coherent quantum state). Ergo, "time arrow" is perfectly meaningless.

That "temporal mess" is due to the prejudice of size and energy being inadequate as to that generalized physical group of action. However, the same seemingly "antiscientific" mess turns out to be ordered very well instantly once one has rejected that prejudice and embraced quantum information conservation: a grandiose and striking harmony appears in both material and spiritual worldview as unified as generalized…

The relation of both Noether theorems interpreted in terms of energy conservation reveals still one symmetry (or idempotency) of the local and global space being complimentary to each other also in quite abstract, set-theoretical meaning:

Each of them is the class of equivalence of the other. So, choosing a certain element (i.e. physically, "state") of the one, the other turns out to be absolutely uncertain therefore implying a principle of uncertainty yet of the level of set theory. The viewpoint of size and energy orders them unambiguously: first, the global space being the macroscopic or megascopic as well as single one; then, infinitely many microscopic local spaces as if "within" any point of the global one.

However, that ordering is due to the usual prejudice of common sense even scientific: in fact, it generates a huge and fundamental mess suggesting for both small and big to be absolute properties rather than mutual relations therefore transforming each into the other in a cyclic perfect symmetry meant in "Yin – Yang" (e.g. in their common and well-known graphic representation) forming "Tao", the pathway of all …

The viewpoint of size and energy, dominating in Europe, in its philosophy and science, destroys the perfect harmony and symmetry, therefore generating a mess, chaos in the universe being "dark" and unrecognizable almost thoroughly, 96 %.

The viewpoint of quantum information as far as it is independent of size and energy allows for the perfectly symmetric, and thus, stable equilibrium, the equilibrium of Yin and Yang to be restored, and particularly, dark matter and dark energy to be recognized as entanglement correlative to gravity. The local space is global to the global space, and vice versa as well.

Indeed, Noether's theorems allow for the prefect symmetry and harmony, in which the global is local neither less nor more than the local is global, to be restored. The bijection of k-number parameters and k-number differential equations after the second theorem can be interpreted as unambiguously associated with each point of the global space and each local space "within it". Then the single bijection of a symmetry (conservation) and a group (already being a *generalized* Lie one as the pair of action and quantum information) is not other than the class of equivalence of k-number elements definable by the second theorem.

The literal approach of the first theorem referring to energy and time, or speaking figuratively, to the "size" and an occasional order of time, keeps the subordination of the local spaces obeying the global space in the second theorem. However, once the case has been already generalized to the "twins" of action and quantum information, the viewpoint "upside down" turns out to be equally valid though complimentary:

The global space(s) is (are) as plural as obeying the local space(s) just as in the usual "size" prejudice of common sense even scientific. In fact, that paradoxical viewpoint to the global space turning out to both plural and obeying the local space is well-known as the many-worlds interpretation of quantum mechanics developed in Hugh Everett III's PhD thesis. There, any global space is a separate or parallel "universe", each generated from a single local space inherent to quantum mechanics.

That revolution is possible only under the condition of suspending the "size and order" viewpoint by that of quantum information. Indeed, the global space as pseudo-Riemannian space suggests many other spaces similar to it, each of which can be interpreted as a parallel universe and all of them originating from a single class of equivalence what the "local" separable complex Hilbert space of quantum mechanics represents for them.

The "four-letter theorem" (Penchev 2020 May 5; Penchev 2020 July 20) can be inferred as a direct corollary from quantum-information conservation:

A very simple proof of the "four-letter theorem" (that the universal alphabet for anything in the universe to be distinguished needs only four letters) embodies the following consideration:

Any qubit teleported needs two additional bits of classical information to be restored unambiguously anywhere in the universe. Those two bits are equivalent to four letters.

The meaning of them after teleportation is the following:

The one bit (two letters) determines between a qubit and its anti-isometric dual counterpart at the source. The other bit (still two different letters) serves analogically at any destination.

In other words, if one qubit be considered as omnipresent anywhere, the four letters are necessary to determine it additionally in an unambiguous way anywhere else.

Since anything is a set of qubits independent of each other in general, any qubit can be considered as a "digit" among an infinite set of digits in the universal natural notation system of the being itself.

Any of those digits (also an infinite set in general) needs four letters for the corresponding "natural number" notating anything by itself to be assigned unambiguously.

Furthermore, it can be interpreted as "showing itself by itself in itself" those as the "phenomenon" (in Husserl and Heidegger) of the thing at issue.

The same four letters (e.g. after teleportation) can be interpreted quite naturally in a temporal way:

The former bit distinguishes the past from the future at the source, the latter bit does the same at any destination.

The equivalence of the "absence of hidden variables" and the "conservation of energy conservation" in quantum mechanics

The close link of the "absence of hidden variables" and the "conservation of energy conservation" was demonstrated already above in a wider context therefore suggesting their mutual implication.

The proof that the former implies the latter, as well as vice versa, can be divided into two statements:

1. The absence of hidden variables implies the conservation of energy conservation in quantum mechanics.

2. The conservation of energy conservation implies the absence of hidden variables in quantum mechanics.

The former can be represented in terms of the system "apparatus – quantum entity" so. The apparatus and the measured elementary particles are absolutely independent of each other. Any dependence would imply a certain "hidden variable" corresponding to that dependence. Then, the state of the quantum system, and particularly its energy, would be invariant to which one exactly among all the possible apparatus measures the quantum system at issue. This means the “conservation of energy conservation” in quantum mechanics.

The latter implication is discussed now. Energy conservation as to any given quantum system means a class of equivalence, each of which can be described unambiguously by the corresponding wave function representing the state of the system in question. This follows directly from the unification of Heisenberg’s matrix mechanics and Schrödinger’s wave mechanics into the separable complex Hilbert space of the unified “quantum mechanics” as its vector interpretation and its function interpretation (Neumann 1932)[5]. Thus, the wave function of any investigated entity is independent of any apparatus, which implies in turn the absence of hidden variables, i.e. just the latter statement.

One has to elucidate immediately the seeming contradiction between the proved equivalence and the experimentally corroborated existence of entanglement implying the irremovable quantum correlations (or “contextuality”) of the apparatus and measured entities in quantum mechanics:

The equivalence at issue means the class of equivalence represented by any wave function, i.e. the state of the measured system as “objective”, and thus fundamentally independent of all the class of possible apparatus (whichever of them be used).

Entanglement means, and thus, distinguishes a certain element among the same class. In other words, all the possible states of entanglements constitutes a class of equivalence meant in the proved mutual implication rather than any of its elements being distinguishable by entanglement. Particularly, the same class can be represented by a special, standard, or “zero” element of zero entanglement, i.e. by the absence entanglement: the exact orthogonality of the corresponding wave functions.

The Standard model involves the identification of the class of equivalence to its “zero” element by the identification of the local and global space as the same separable complex Hilbert space since its infinite dimensionality allows for any “curvature” (i.e. entanglement) to be “flatten” equivalently, being due to the infinite “extendibility” for its infinite dimensionality. Thus, the Standard model fundamentally excludes as any theory of quantum gravity as any theory of entanglement being able to substitute any of them with its absence (rather paradoxically, but only at first glance).

On the contrary, general relativity excludes fundamentally the identification of all the elements in that class of equivalence as different space-time trajectories and different tensors of energy-momenta in any point of each of them in general.

Thus, quantum mechanics and general relativity see always the two complementary “sides” of each mechanical phenomenon, which are complimentary even still in terms of set theory: quantum mechanics discusses the class of equivalence as an ordinal number, and general relativity adds the distinction of all the well-orderings within each of those ordinal numbers. The viewpoint of general relativity is more detailed, but that of quantum mechanics is more general: general relativity “sees the trees” being fundamentally unable to “see the forest”; and quantum mechanics, on the contrary. So, both are complimentary to each other, and thus, both are equally true and false simultaneously.

[5] The history of the unification of matrix mechanics and wave mechanics into quantum mechanics is meant according to Neumann (1932) in the present paper thoroughly. The only focus of the consideration is concentrated on the way for the well-ordered vectors of matrix mechanics to be consistent to the fundamentally unorderable coherent states by wave functions meant in undulatory mechanics as postulated in quantum mechanics

Anyway, one can introduce the “transcendental viewpoint” to unify them (as in this paper). It implies the viewpoint of quantum information and its conservation as well as the “crazy” worldview about the universe within a single quantum furthermore represented as all the quanta within the universe, fortunately, “crazy” only as to common science rather than as to logic and experimental science. The “big” and “small” are able to share and conserve quantum information, and then, the “size” (or energy) does not matter at all.

Once the viewpoint of quantum information is granted, one can demonstrate how its conservation implies both “absence of hidden variables” and “conservation of energy conservation” in quantum mechanics (but not vice versa, for general relativity is not involved). Here is how:

The conservation of quantum information is equivalent to the “conservation of wave function” in the following meaning. The unambiguously certain wave function of a state of a quantum system is the same in any point of space-time as well as being measured by any possible apparatus. Speaking figuratively, it is omnipresent, eternal and unchangeable. Furthermore, it is the class of equivalence of all corresponding well-ordered vectors, each of which can be obtained by any other by a certain unitary transformation, and thus, sharing the same energy. Briefly, the conservation of quantum information implies energy conservation in quantum mechanics (which cannot be deduced from the principle of least action since, for example, any quantum discreteness of time is inconsistent to its well-ordering, or “arrow”). The conservation of energy conservation is equivalent to the absence of hidden variables in quantum mechanics as it is demonstrated above. Consequently, quantum information conservation implies also the absence of hidden variables.

Anyway, the last statement may be inferred directly. Any “hidden variable” would contradict the “conservation of wave function” defined in the previous paragraph. So, *modus tollens* implies the absence of hidden variables under the condition of quantum information conservation. Indeed, any “hidden variable” would imply and would mean a certain dependence of the wave function of a certain entity in the universe different than the state of the quantum system at issue, and thus, its changeability in principle.

A direct corollary from quantum information conservation is that only quantum information is shared by the apparatus and measured entity in quantum mechanics. The energy (as well as any other classical physical quantity) exchanged between them is arbitrarily and irrelevant to the conserved shared quantum information. Thus, it can be granted as zero (as in the Standard model) to be simplified both notations and calculations.

The same postulation, however, closes the pathway to any theory of both entanglement and quantum gravity since they both consist in an exactly determined equivalent amount of as mass (for space-time tensor) as energy (for the tensor of energy-momenta) choosing a certain state of entanglement among all the class of equivalence meant implicitly by the Standard model.

“Mach’s principle” in general relativity (Einstein 1918) implies an equivalent quantities of mass or energy for any certain state of entanglement being properly quantum information only (as far as neither time nor space are necessary conditions for it and unlike any classical entity). However, entanglement on the “screen” of space-time is being projected (if one grants “Mach’s principle”) just as gravity in the way being described by general relativity. Therefore, entanglement would be available in the Einstein filed equation as a space-time field of the “cosmological constant”, which would turn out to be different in any space-time point in general. That equivalent space-time of entanglement (visualizable, for example, as a field of the cosmological constant in general relativity) is the real source of both dark matter and dark energy. Thus, about 96% of the universe is due to quantum information directly as entanglement, furthermore being fundamentally unrecognizable in the framework of the Standard model (as it is explained above).

The universe contains physical objects quite different in their size, or accordingly, in their energy. The standard viewpoint of energy conservation considers them as rather different according to their physical influence to each other. Energy stratifies the universe into successive hierarchical levels. Their rigorous subordination is the way of the universe considered as an energetic entity to be a whole. A necessary condition of that picture is the single continuous time unifying the universe not less than its Noether correlate, energy.

As far as quantum mechanics grants energy conservation, it is able to share the same picture, in which the energetically microscopic "cognitive niches" are predetermined for it. However, it is no able to share the correlative dogma of the single continuous time unifying the universe. Pauli's way out is time not to be a quantum quantity sharing a Hermitian operator as all others, but only a microscopic quantity, "only a number rather than an operator". At the cost of the victim of the quantum time, the unified picture of the universe subordinated and hierarchized in size and energy could be kept … however until the time when both "energy matter" and "dark energy" were discovered, the beginning of our century; and even much worse, they turned out to be 96% of the mass and energy of the universe.

Thus, that picture of the universe hierarchized and subordinated in size and energy is absolutely wrong, a superstition much more inadequate than adequate.

By taking off the "confusing and distorting glass" of energy conservation and replacing it by quantum information conservation, one observes a quite different universe, in which the unity is due to exchanging or more precisely, to sharing the same quantum information rather than to the subordination and hierarchy in size and energy. This is a "democratic universe", in which the big and small are equal "politically" and in physical influence. That universe is rather a fluid medium rather than a crystal, rather an Internet medium exchanging information though quantum than a bureaucratic state with frozen institutions …

As it is well-known, the idea of entanglement is suggested[6] by Einstein, Poldolsky, and Rosen (1935) as "reduction ad absurdum" to demonstrate the ostensible "incompleteness of quantum mechanics". A problem rather philosophical than mathematical and physical is whether one can consider any certain state of entanglement as a corresponding values of "hidden variables" (though nonlocal in the final analysis).

As it is elucidated in detail above, quantum mechanics means the class of equivalence of all the well-orderings referring to the same ordinal as a single state and representable by a single wave function accordingly. Entanglement introduces (at least) still one wave function necessarily non-orthogonal to the former one, therefore, choosing a certain well-ordering (or a subset of well-

[6] The same idea is suggested independently in the same year by Schrödinger (1935) in a paper about the then philosophy of quantum mechanics, however, not as an absurd idea, not as a proof for the incompleteness of quantum mechanics, and calling for "some spooky action in a distance" in physics, but as an absolutely real phenomenon for research. Nonetheless, the "triple article" of Einstein, Podolsky, and Rosen (in fact, rejecting entanglement) is much more famous (undeservedly).

orderings) among the former class of equivalence unambiguously determined by the former wave function. That one single element or subset can be defined furthermore as the nonzero intersection of the classes of equivalences corresponding to each of the two (or more) wave functions being entangled.

From a logical viewpoint, quantum mechanics studies only properties of quantum states (i.e. wave functions), and entanglement is some relation of quantum states (thus needing more than one wave function to be determined unambiguously. However, quantum mechanics means only the class of equivalence of all relations of that kind, referring to a certain property of that kind. This allows for it to ignore all states of entanglement meaning each of them as a special new property (wave function) of a new quantum system consisting of all corresponding entangled states (wave functions).

This "right and freedom" of quantum mechanics to ignore entanglement absolutely, thoroughly and fundamentally is quite legal and even "written in its constitution" in three different "paragraphs", namely in: (1) Bohr's postulate about what quantum mechanics studies; (2) the separable complex Hilbert space as the fundamental mathematical formalism of quantum mechanics; (3) the infinite dimensionality of that Hilbert space in general (being able to "flatten" any "curvature" due to entanglement, or to constitute a new whole of any quantum states entangled or not).

Thus, entanglement turns out to be irrelevant to the problem of "hidden variables" as far as their absence is a direct corollary from its "written constitution" as well as its completeness.

The situation, however, is fundamentally different in pseudo-Riemannian space being both finitely dimensional and limited until the present moment in time (which is a certain one, and thus, finite rather than infinite. So, the "constitutional completeness" of quantum mechanics (as well as the fundamental absence of hidden variables) turns out to be inapplicable to general relativity. Figuratively speaking, the force of gravity and its field appears to compensate the difference between the finiteness of general relativity to the infinity of quantum mechanics, therefore, eider excluding quantum gravity as a "mistake in definition" fundamentally, or equating it to the "classical" smooth field of gravity in general relativity (in virtue of "transcendental field" as above).

Even a philosophical reflection in terms of "a whole" and its parts" is possible to elucidate fundamentally the difference in the approaches of quantum mechanics and general relativity, and the way for gravity to appear as if "in-between, between the classical (where it is) and quantum (where it is not)":

Quantum mechanics introduces a new viewpoint in physics: "outside", i.e. outside of the investigated system. That viewpoint is not fundamentally new as far this is the viewpoint of phenomenological thermodynamics. However, it discusses almost only specific thermodynamic quantities referring to the system as a whole and inapplicable to any single element of it. The exception is energy, and its conservation transfers between a thermodynamic consideration and a mechanic one, The Boltzmann statistic thermodynamic is the correlate of phenomenological thermodynamics if one change the viewpoint from "outside of the system" into "inside of it", after which all phenomenological thermodynamic quantities turn out to be probability distributions of huge statistic ensembles of elements such as atoms, molecules (as well as their quantum equivalents as fermions, bosons), etc.

Gibbs thermodynamics demonstrates that the viewpoint inside of the system is not a necessary condition for a statistic consideration. Thus, it can be suggested to be a generalization or modification of statistic thermodynamic sharing that of phenomenological thermodynamics, i.e. "outside". It substitutes Boltzmann's elements with Gibbs's states of the system.

Quantum mechanics can be considered as another generalization of both statistic and phenomenological thermodynamics, that generalization meaning only a single element or a few ones, which is possible just for the fundamental Planck constant, initially (as to the radiation of the "absolutely black body") and until now allowing for an even only thermodynamic interpretation. However, the properly statistic viewpoint in Boltzmann's manner seems to be a nonsense just for the Planck constant. Anyway, one can introduce Gibbs's viewpoint to quantum mechanics very easily considering any quantum state as a Gibbs state, and the corresponding wave function as a characteristic function of a probability (density) distribution of quantum states inferring the "principle of superposition" from that, particularly. Further, one can construct a Boltzmann correlate "inside" once its Gibbs counterpart is granted already, therefore and eventually adding a special force field (such as gravitational one) to equate both viewpoints to each other.

The consideration above has demonstrated already that gravitational field appears to equate the "actually infinite viewpoint" of quantum mechanics to the finite one of classical mechanics. Furthermore, the same equating compensation can be interpreted for the eventual difference between the Boltzmann viewpoint ("inside") and the Gibbs one ("outside") as to quantum mechanics interpreted as generalized thermodynamic theory referring to single elements.

At least, it can be interpreted only philosophically, in terms of a whole and its parts, or only mathematically, in terms of a set (class) and its subsets (subclasses). The standard viewpoint of quantum mechanics (implying the conservation of energy conservation, the absence of hidden variables, and thus, reducing entangled subsystems, to a non-entangled whole) means philosophically the external viewpoint to a whole or a set or class. Actual infinity is involved necessarily to supply the standard viewpoint of quantum mechanics. Entanglement is only an auxiliary, practical or even only technical approach in its framework, always removable from a fundamental viewpoint. It is due to involving the auxiliary viewpoint of parts of a quantum system, therefore not being necessary in general and immediately vanishing "into thin air" after restoring the standard viewpoint of a single quantum whole. The absence of hidden variables (including nonlocal ones due to entanglement) and the conservation of energy conservation are direct corollaries then. Mach's principle in general relativity, particularly stating that quantum information cannot be a source of gravitational field is a natural counterpart of them. Main problems of that "standard quantum mechanics" (embodied further in the Standard model) are dark matter and dark energy as well as the missing "quantum gravity".

On the contrary, the "nonstandard quantum mechanics", whic postulates (or proves) quantum information conservation, infers the absence of hidden variables and energy conservation from it. It admits the violation of energy conservation where the concepts of energy and time are not directly applicable in the same way as in classical mechanics (the case of quantum mechanics and information), demonstrates that entanglement is a correlate of gravitational filed therefore violating Mach's principle and explaining as dark matter as dark energy as well as the fundamental impossibility of "quantum gravity" in the framework of the standard quantum mechanics. All those three huge anomalies in it as a theory are due to the "distorting glasses" of energy conservation not allowing for quantum information to be seen, understood and observed experimentally as the most fundamental physical substance underlying the two ones well-known until now: energy and mass. A cyclic worldview to the "universe within a single quantum" and the rejecting picture of the universe as a hierarchy subordinated in size and energy are also inherited features of the same fundamental change of the paradigm, to which the present paper belongs, particularly.

Is quantum mechanics complete or incomplete in the final analysis anyway?

Well, it is complete in definition, but the viewpoint to it as incomplete is much more interesting, heuristic, and fruitful …

Conclusions & future work

Energy conservation is problematic in quantum mechanics. This is due to two reasons. It cannot be inferred from the principle of least action for it is invalid[7] in quantum mechanics. The "fourth" (energy to time) uncertainty implies the violation of energy conservation in general.

Anyway, energy conservation is conserved in quantum mechanics and even postulated in the mathematical base of quantum mechanics, the separable complex Hilbert space. However, the reasons for this are absolutely different from those in classical mechanics. They are contained in equating Heisenberg's matrix mechanics and Schrödinger's wave mechanics as the two equivalent interpretations (as vectors and functions) of the separable complex Hilbert space of quantum mechanics nowadays, namely in the unitarity implied by that equation.

The conservation of energy conservation is equivalent to the absence of hidden variables in quantum mechanics as well as analogical to "Mach's principle" in general relativity, postulating only mass and energy as possible sources of gravitational field. On the contrary, entanglement (or quantum information) can be considered as an alternative source of gravitational field along with energy conservation and therefore as a possible reason of dark matter and dark energy, and thus, involving any state of entanglement as an auxiliary, but nonlocal hidden variable.

Furthermore, it can be generalized as the invariance to choice/ ordering due to the equivalence of the well-ordering "theorem" and the axiom of choice. One can introduce generalized reference frames (discrete, quantum, or out of the observed system) as well as the corresponding relativity designable as quantum and consisting in the former invariance. There exists the corresponding parallel generalization in quantum mechanics (called in the paper "nonstandard quantum mechanics") postulating studying the apparatus by the any possible quantum system measurable by it instead of a certain quantum by any possible apparatus able to measure it (as the usual "standard quantum mechanics" does).

After that consideration also meaning the above quantum invariance of the discrete and quantum, one can generalizes the approach of the Standard model identifying the "local" and "global" space (involved and defined by it) as the same single separable complex Hilbert space of the standard quantum mechanics. The generalization identifies that Hilbert space as both local and global (as in the Standard model) with pseudo-Riemannian space of general relativity under the condition of quantum invariance (as above), and possibly, after the consideration and viewpoint involved by the nonstandard quantum mechanics (as if that of the apparatus). The identification newly of both global and local space serves to be defined a field and space both called transcendental. It allows for entanglement to be also identified rigorously and mathematically, and furthermore, dark matter and dark energy to be explained (once Mach's principle is granted) as being due to the physical action of quantum information (respectively, entanglement).

[7] That invalidity is obvious after the Feynman interpretation of quantum mechanics.

The same generalization implies a few essential changes in the physical and philosophical worldview: quantum information is to be heralded as the most fundamental substance of the universe therefore underlying the known until now: mass and energy. Quantum information conservation is the most fundamental law of conservation. The worldview of the universe both hierarchized and subordinated in size and energy is to be replaced by the picture of the cyclic universe as if situated in a single quantum. It wholeness is supported by the omnipresent quantum information as if coordinating and synchronizing instantly all parts of it rather than by the subordinated hierarchy in energy and size being inessential and auxiliary.

Particularly, a thermodynamic, entropic and informational interpretation of general relativity is a direct corollary from the transcendental field as it is introduced in the present paper. A next paper intends to demonstrate the equivalence of general relativity and Verlinde's theory of gravitation on that base eventually under certain conditions formulated explicitly.

References:

Aspect, A., Grangier, R, Roger, G. (1981) "Experimental Tests of Realistic Local Theories via Bell's Theorem," *Physical Review Letters* **47** (7): 460-463.

Aspect, A., Grangier, R, Roger, G. (1982) "Experimental Realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment: A New Violation of Bell's Inequalities," *Physical Review Letters* **49** (2): 91-94.

Bell, J. (1964) "On the Einstein Podolsky Rosen paradox,“ *Physics* **1** (3): 195-200.

Bohr, N. (1935) "Can Quantum-Mechanical Description of Physical Reality be Considered Complete?" *Physical Review* **48** (8): 696-702.

Bohr, N., Kramers, H.A., Slatter, J.A. (1924) "The quantum theory of radiation," *Philosophical Magazine and Journal of Science* **47** (281, May 1924): 785-802.

Clauser, J. Horne, M., Shimony, A., Holt, R. (1969) "Proposed Experiment to Test Local Hidden-Variable Theories," *Physical Review Letters* **23** (15): 880-884.

Clauser, J, Horne, M. (1974) "Experimental consequences of objective local theories," *Physical Review D* **10** (2): 526-535.

Einstein, A. (1918) "Prinzipielles zur allgemeinen Relativitätstheorie," *Annalen der Physik* **55** (4): 241-244.

Einstein, A., Podolski, B., Rosen, N. (1935) "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" *Physical Review* **47**: 777-780.

Kochen, S, Specker, E. (1968) "The Problem of Hidden Variables in Quantum Mechanics," *Journal of Mathematics and Mechanics* **17** (1): 59-87.

Neumann, J. (1932) *Grundlagen of Quantenmechanik*, Berlin, Springer.

Penchev, V. D. (2020, May 5). From the four-color theorem to a generalizing "four-letter theorem": A sketch for "human proof" and the philosophical interpretation. https://doi.org/10.31235/osf.io/yf5x7

Penchev, V. D. (2020, July 20). The relationship of arithmetic as two twin Peano arithmetic (s) and set theory: A new glance from the theory of information. https://doi.org/10.31235/osf.io/ehbtn

Schrödinger, E. (1935) "Die gegenwärtige Situation in der Quantenmechanik," *Die Naturwissenschaften* **23** (48): 807-812; **23** (49): 823-828; **23** (50): 844-849.